

NAG Fortran Library Routine Document

F04LEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F04LEF solves a system of tridiagonal equations following the factorization by F01LEF. This routine is intended for applications such as inverse iteration as well as straightforward linear equation applications.

2 Specification

```
SUBROUTINE F04LEF (JOB, N, A, B, C, D, IPIV, Y, TOL, IFAIL)
  INTEGER          JOB, N, IPIV(N), IFAIL
  double precision A(N), B(N), C(N), D(N), Y(N), TOL
```

3 Description

Following the factorization of the n by n tridiagonal matrix $(T - \lambda I)$ as

$$T - \lambda I = PLU$$

by F01LEF, F04LEF may be used to solve any of the equations

$$(T - \lambda I)x = y, \quad (T - \lambda I)^T x = y, \quad Ux = y$$

for x , the choice of equation being controlled by the parameter JOB. In each case there is an option to perturb zero or very small diagonal elements of U , this option being intended for use in applications such as inverse iteration.

4 References

Wilkinson J H (1965) *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

Wilkinson J H and Reinsch C (1971) *Handbook for Automatic Computation II, Linear Algebra* Springer-Verlag

5 Parameters

1: JOB – INTEGER *Input*

On entry: must specify the equations to be solved.

JOB = 1

The equations $(T - \lambda I)x = y$ are to be solved, but diagonal elements of U are not to be perturbed.

JOB = -1

The equations $(T - \lambda I)x = y$ are to be solved and, if overflow would otherwise occur, diagonal elements of U are to be perturbed. See parameter TOL.

JOB = 2

The equations $(T - \lambda I)^T x = y$ are to be solved, but diagonal elements of U are not to be perturbed.

JOB = -2

The equations $(T - \lambda I)^T x = y$ are to be solved and, if overflow would otherwise occur, diagonal elements of U are to be perturbed. See parameter TOL.

JOB = 3

The equations $Ux = y$ are to be solved, but diagonal elements of U are not to be perturbed.

JOB = -3

The equations $Ux = y$ are to be solved and, if overflow would otherwise occur, diagonal elements of U are to be perturbed. See parameter TOL.

- 2: N – INTEGER *Input*
On entry: n , the order of the matrix T .
Constraint: $N \geq 1$.
- 3: A(N) – *double precision* array *Input*
On entry: the diagonal elements of U as returned by F01LEF.
- 4: B(N) – *double precision* array *Input*
On entry: the elements of the first superdiagonal of U as returned by F01LEF.
- 5: C(N) – *double precision* array *Input*
On entry: the subdiagonal elements of L as returned by F01LEF.
- 6: D(N) – *double precision* array *Input*
On entry: the elements of the second superdiagonal of U as returned by F01LEF.
- 7: IPIV(N) – INTEGER array *Input*
On entry: details of the matrix P as returned by F01LEF.
- 8: Y(N) – *double precision* array *Input/Output*
On entry: the right-hand side vector y .
On exit: is overwritten by the solution vector x .
- 9: TOL – *double precision* *Input/Output*
On entry: the minimum perturbation to be made to very small diagonal elements of U . TOL is only referenced when JOB is negative. TOL should normally be chosen as about $\epsilon \|U\|$, where ϵ is the *machine precision*, but if TOL is supplied as non-positive, then it is reset to $\epsilon \max |u_{ij}|$.
On exit: if on entry TOL is non-positive, it is reset as just described. Otherwise TOL is unchanged.
- 10: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Chapter P01 for details.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry, $N < 1$,
or $JOB = 0$,
or $JOB < -3$ or $JOB > 3$.

$IFAIL > 1$

Overflow would occur when computing the $(IFAIL - 1)$ th element of the solution vector x . This can only occur when JOB is supplied as positive and either means that a diagonal element of U is very small or that elements of the right-hand side vector y are very large.

7 Accuracy

The computed solution of the equations $(T - \lambda I)x = y$, say \bar{x} , will satisfy an equation of the form

$$(T - \lambda I + E)\bar{x} = y,$$

where E can be expected to satisfy a bound of the form

$$\|E\| \leq \alpha \epsilon \|T - \lambda I\|,$$

α being a modest constant and ϵ being the *machine precision*. The computed solution of the equations $(T - \lambda I)^T x = y$ and $Ux = y$ will satisfy similar results. The above result implies that the relative error in \bar{x} satisfies

$$\frac{\|\bar{x} - x\|}{\|\bar{x}\|} \leq c(T - \lambda I) \alpha \epsilon,$$

where $c(T - \lambda I)$ is the condition number of $(T - \lambda I)$ with respect to inversion. Thus if $(T - \lambda I)$ is nearly singular, \bar{x} can be expected to have a large relative error. Note that F01LEF incorporates a test for near singularity.

8 Further Comments

The time taken by F04LEF is approximately proportional to n .

If you have single systems of tridiagonal equations to solve you are advised that F07CAF (DGTSV) requires less storage and will normally be faster than the combination of F01LEF and F04LEF, but F07CAF (DGTSV) does not incorporate a test for near singularity.

9 Example

This example solves the two sets of tridiagonal equations

$$Tx = y \quad \text{and} \quad T^T x = y$$

where

$$T = \begin{pmatrix} 3.0 & 2.1 & 0 & 0 & 0 \\ 3.4 & 2.3 & -1.0 & 0 & 0 \\ 0 & 3.6 & -5.0 & 1.9 & 0 \\ 0 & 0 & 7.0 & -0.9 & 8.0 \\ 0 & 0 & 0 & -6.0 & 7.1 \end{pmatrix} \quad \text{and} \quad y = \begin{pmatrix} 2.7 \\ -0.5 \\ 2.6 \\ 0.6 \\ 2.7 \end{pmatrix}.$$

The example program first calls F01LEF to factorize T and then two calls are made to F04LEF, one to solve $Tx = y$ and the second to solve $T^T x = y$.

9.1 Program Text

```

*   F04LEF Example Program Text
*   Mark 14 Revised. NAG Copyright 1989.
*   .. Parameters ..
INTEGER          NMAX
PARAMETER        (NMAX=50)
INTEGER          NIN, NOUT
PARAMETER        (NIN=5,NOUT=6)
*   .. Local Scalars ..
DOUBLE PRECISION LAMBDA, TOL
INTEGER          I, IFAIL, JOB, N
*   .. Local Arrays ..
DOUBLE PRECISION A(NMAX), B(NMAX), C(NMAX), D(NMAX), Y(NMAX),
+               Z(NMAX)
INTEGER          IN(NMAX)
*   .. External Subroutines ..
EXTERNAL         F01LEF, F04LEF
*   .. Executable Statements ..
WRITE (NOUT,*) 'F04LEF Example Program Results'
*   Skip heading in data file
READ (NIN,*)
READ (NIN,*) N
WRITE (NOUT,*)
IF (N.LT.1 .OR. N.GT.NMAX) THEN
    WRITE (NOUT,99999) 'N is out of range. N = ', N
ELSE
    READ (NIN,*) (A(I),I=1,N)
    IF (N.GT.1) THEN
        READ (NIN,*) (B(I),I=2,N)
        READ (NIN,*) (C(I),I=2,N)
    END IF
    TOL = 5.0D-5
    LAMBDA = 0.0D0
    IFAIL = 1

*
    CALL F01LEF(N,A,LAMBDA,B,C,TOL,D,IN,IFAIL)
*
    IF (IFAIL.NE.0) THEN
        WRITE (NOUT,*)
        WRITE (NOUT,99999) 'F01LEF fails. IFAIL =', IFAIL
    ELSE
        IF (IN(N).NE.0) THEN
            WRITE (NOUT,*) 'Matrix is singular or nearly singular'
            WRITE (NOUT,99998) 'Diagonal element', IN(N), 'is small'
        ELSE
            READ (NIN,*) (Y(I),I=1,N)
            DO 20 I = 1, N
                Z(I) = Y(I)
20          CONTINUE
            JOB = 1
            IFAIL = 1

*
            CALL F04LEF(JOB,N,A,B,C,D,IN,Y,TOL,IFAIL)
*
            IF (IFAIL.NE.0) THEN
                WRITE (NOUT,*)
                WRITE (NOUT,99999) 'F04LEF fails. IFAIL =', IFAIL
            ELSE
                WRITE (NOUT,*)
                WRITE (NOUT,*) 'Solution vector for T*X = Y'
                WRITE (NOUT,99997) (Y(I),I=1,N)
            END IF
            JOB = 2
            IFAIL = 1

*
            CALL F04LEF(JOB,N,A,B,C,D,IN,Z,TOL,IFAIL)
*
            IF (IFAIL.NE.0) THEN
                WRITE (NOUT,*)
                WRITE (NOUT,99999) 'F04LEF fails. IFAIL =', IFAIL

```

```

                ELSE
                WRITE (NOUT,*)
                WRITE (NOUT,*)
+                'Solution vector for transpose(T)*X = Y'
                WRITE (NOUT,99997) (Z(I),I=1,N)
                END IF
            END IF
        END IF
    END IF
    STOP
*
99999 FORMAT (1X,A,I5)
99998 FORMAT (1X,A,I4,A)
99997 FORMAT (1X,5F9.3)
    END

```

9.2 Program Data

F04LEF Example Program Data

```

5
3.0  2.3 -5.0 -0.9  7.1
2.1 -1.0  1.9  8.0
3.4  3.6  7.0 -6.0
2.7 -0.5  2.6  0.6  2.7

```

9.3 Program Results

F04LEF Example Program Results

```

Solution vector for T*X = Y
-4.000  7.000  3.000 -4.000 -3.000

```

```

Solution vector for transpose(T)*X = Y
-4.630  4.880 -0.555  0.672 -0.377

```
